Asymmetric Threats Modeling and Application of LINGO Language

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The latest investigations in war gaming area are focused on new approaches to asymmetric threat modeling, analysis and prediction. Contemporary developments in game theory provide a flexible and powerful framework to model adversarial motivation and to generate credible asymmetric strategies for improved automation of behaviors in simulations and to support operations analysis and planning.

In most decision making situations the profits and losses are determined not only our decisions, but by the outside forces. The standard terminology applied to the problem to be considered is the game theory, especially through the features of optimization modeling with programming languages (LINGO [1]).

ASYMMETRIC THREATS

The notion, "asymmetric", as applied to asymmetric threat or asymmetric warfare has several meanings. Fundamentally, asymmetry upsets the offensive/defensive equilibrium to the perpetrator's perceived advantage by exploiting defense vulnerabilities or offense restraints with unconventional methods. An asymmetric attack is much less expensive to conduct than it is to defend against. Conversely, it is more difficult (expensive) to perceive an asymmetric defense tactic than it is to set one up.

"In an age when national decision making and commitment is driven more by public opinion than by policy principles and leadership we are particularly vulnerable to enemy information operations (IO) and propaganda which are generally considered to be tools in the asymmetric war chest"[2]. In consequence, modern asymmetric conflict tends to simultaneously expand the dimension of the conflict and merge decisions and actions conventionally separated into strategic, operational and tactical categories. The acquisition, operation and maintenance of the Command, Control, Communications, Computers, and Intelligence (C4I) infrastructure open yet another possibility for asymmetric attack by embedding or integrating commercial off the shelf (COTS) technology.

Asymmetric targeting is yet another dimension. Terrorism often intentionally strikes civilian or other non-combatant targets of opportunity, for the purpose of creating panic and shaking confidence in the competence of the authority or damaging the social stability and welfare.

The main feature of the asymmetric threats is the use of the scanty (inadequate) position advantages. Usually there are actions based on the unconventional ways and means, i.e. they don't oppose a force to the

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exceeding force in condition of an untraditional conflict. Such kinds of conflicts are terrorist acts and guerrilla warfare in which the goal is not to win, but to obstruct the powerful adversary to win, i.e. to avoid the loss.

Obviously, the answer does not comprise in military force engagements alone. Wargames are used for both training and analysis as well as in mission planning and rehearsal. The purpose is to synthesize sophisticated and agile C2 decision-making models for wargaming the asymmetric environment.

WAR GAMING

Contemporary strategy and doctrine are based on the joint and coalition operations. Operational wargames typically consist of multi-echelon (blue) participants as main forces, enemy (red), control staff white), and a number of neutral, friendly and coalition teams. Depending on the purposes – training, analysis, rehearsal, etc. – and size, and resolution of the wargame the infrastructure may be involved as well.

Virtual simulations are used in training and exercise wargames to stimulate the C2 equipment of trainees actually in the field, significantly augmenting the training environment with synthetic red or blue forces as needed. The need for valid and realistic simulated component behaviour has long required labor intensive scenario development and setup and, depending on scope, a sizable support team to steer or correct simulation behaviours that have gone off-track during the course of the run. High-resolution, multi-echelon constructive simulations are applied to create authentic and accurate representation of the environment and forces behaviour.

The war games need to incorporate behaviors and combined effects of both major and minor nation states as well as a host of non-state, non-governmental organizations, trans-national and international terrorist organizations operating in the asymmetric environment as well as corporate and criminal entities with significant business interests.

Recently the Operations Other Than War (Peace Making, Peace Keeping, Humanitarian Relief) are of special interest and they represent a natural match between game theory and the asymmetric environment. They create many challenges to the applied game theory in analysis and prediction.

GAME THEORY MODELS

The operation analysis and planning process is based on the ideas of game theory – the mathematical theory of decision-making in conflict situations. The game theory approaches give the opportunity to model the most important elements of the planning process – the analysis of opposing courses of actions, the behavior of the sides, payoffs and losses. These ideas assist to improve and refine the multi-player models with respect to the asymmetric threat.

Classification

There are many classes games, which are differentiated by various criteria – the players' number, number of steps, type of cost function, etc. (Figure 1).

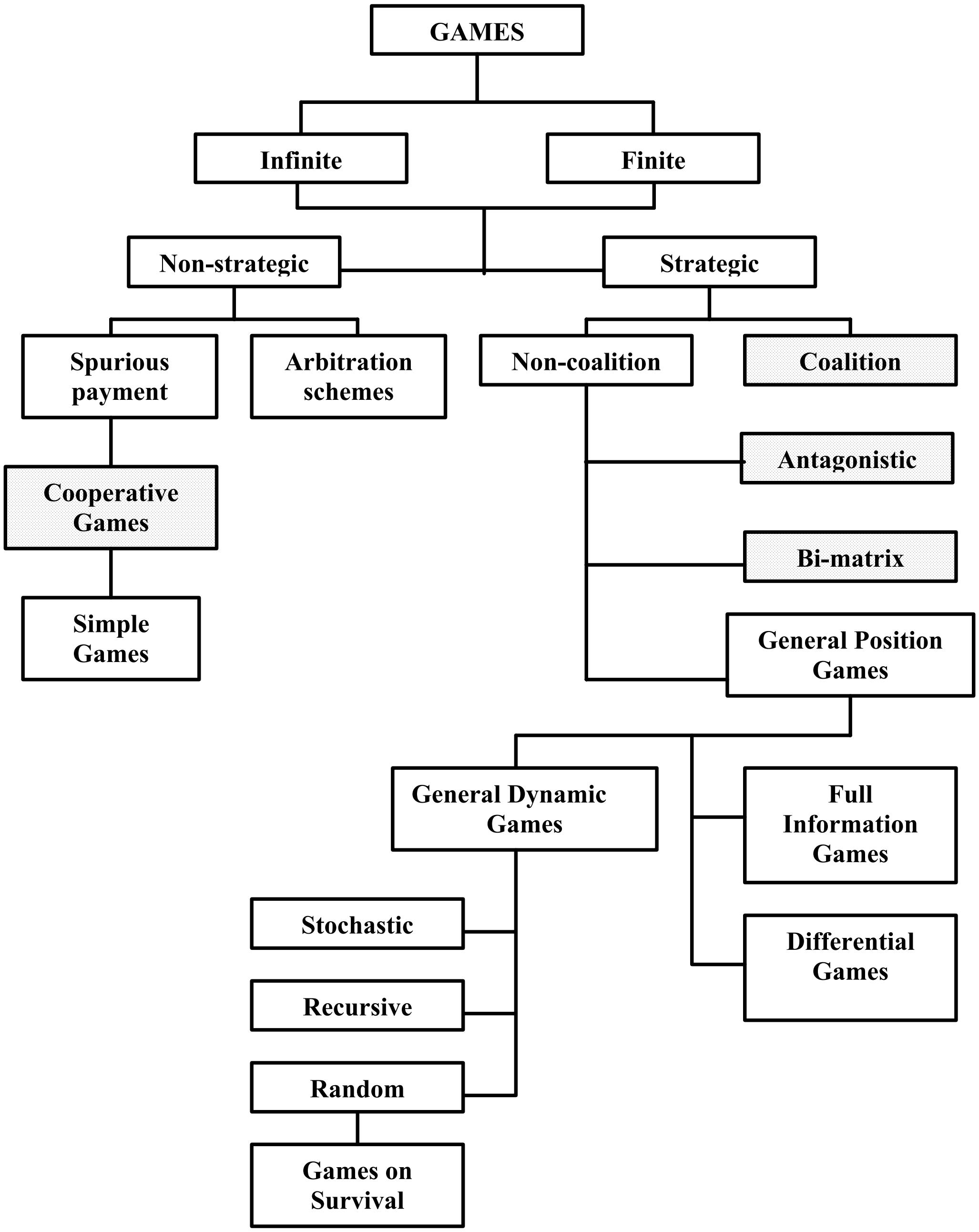

Figure 1

The real conflict can be modeled by the *finite antagonistic game* in case of the following conditions:

1) The conflict is determined by antagonistic interaction of two parties, each of which disposes only final number of possible (probable) actions.

2) The actions of the parties, undertake separately from each other, i.e. each of them has no the information on operation made by other party. The result of these actions is valued by a real number, which determines usefulness of the situation for one of the parties.

3) Each of the parties values both for itself, and for the opponent usefulness of any possible (probable) situation, which can develop as a result of their interaction.

4) The actions of the parties have not formal features. Thus the parties' actions can be treated as abstract homogeneous sets.

If the conflict corresponds to (1-4), defining one of the parties by the player *I* and another – player *II*, we can describe it by the antagonistic game [3]

$$\Gamma = < X, Y, H >, \tag{1}$$

where – X is set of the pure strategies the player I, $X = \{ X_1, X_2 \dots X_m \}$;

Y is set of the pure strategies of the player II, $Y = \{ Y_1, Y_2 \dots Y_n \}$;

H is the function of usefulness of the player I (profit of the player I), which is determined for all couples of possible actions of the players.

The matrix is game-theoretic model of real conflicts adequate on conditions (1-4), i.e. we assign the finite antagonistic game as a matrix:

$$H = \|h_{ij}\|, \qquad h_{ij} = H(i,j), \qquad 1 \le i \le m\,,\ 1 \le j \le n; \tag{2}$$

In order to find a stable optimal strategy it is necessary to solve the following equations:

$$E_1(X, y_j)\sum_{i=1}^{n} h_{ij}x_i = const(j = 1,\dots m); \tag{3}$$

$$E_2(x_i, Y)\sum_{j=1}^{m} h_{ij}y_j = const(i = 1,\dots n);$$

$$\sum_{i=1}^{n} x_i = 1; \tag{4}$$

$$\sum_{j=1}^{m} y_j = 1;$$

Thus the game payoff is:

$$E(X,Y) = \sum_{j=1}^{m} \sum_{i=1}^{n} h_{ij}x_i y_j \tag{5}$$

The strategies $X^* \in X$ and $Y^* \in Y$ are optimal mixed strategies of player *I* and *II*, if the following expression is true:

$E\ (X, Y^*) \le E(X^*, Y^*) \le E(X^*, Y)$ in Cartesian product of the (X, Y).

The solution is in the following form:

$$\begin{aligned} &\|X^*, Y^*, v\| \\ &v = E(X^*, Y^*) \end{aligned} \qquad (6)$$

where v is the game cost.

It is possible to select real conflicts with the reasonably functioning parties and phenomenon, in which exists the undetermination caused by combination of objectively present circumstances – for example conflicts with a nature, or terrorist attack. The simulation of similar phenomena results in the special class of matrix games, namely "matrix games against the nature". Special feature of these conflicts is the impossibility of physical implementation of mixed strategy that requires the random choice of pure strategy. The model is the matrix H:

$$H = \begin{pmatrix} p_1 & r_{12} + p_2 & \dots & r_{1n} + p_n \\ r_{21} + p_1 & p_2 & \dots & r_{2n} + p_n \\ \dots & \dots & \dots & \dots \\ r_{n1} + p_1 & r_{n2} + p_2 & \dots & p_n \end{pmatrix} \qquad (7)$$

where – $p_j > 0$ are the salvage charges (j-strategy of player II) and

$r_{ij} > 0$ are the losses of player II, caused by i-action of the "nature"(player I) in case of j-strategy of player II.

Two-player constant sum game (3) is suitable for modeling the antagonistic collisions. Ordinary linear programming can be used to solve this kind of games.

But there are many situations when the conflict is not strictly antagonistic. The appropriate simulation in these cases is non-constant sum games involving two or more players. There are some styles for creating non-constant sum games. If we restrict the examination to two players, then bi-matrix model extends the methods for two-player constant sum games to non-constant sum games.

Linear programming cannot be used to solve this game. However, closely related algorithms – linear complementary algorithms – are commonly applied. The cost matrix consists of two players' matrixes:

$$A = \|a_{ij}\|, \quad a_{ij} = A(i,j), \qquad (8)$$

$$B = \|b_{ij}\|, \quad b_{ij} = B(i,j),$$

$$1 \le i \le m, \; 1 \le j \le n;$$

where a_{ij} and b_{ij} are respectively the profits of player I and II in situation (i,j).

The probability player *I* to choose alternative *i* is X_i, respectively – for player *II* to choose alternative *j* is *Yj*. Because of this, the following constraints are correct:

$$X_i \geq 0; \qquad \sum_{i=1}^{n} X_i = 1; \tag{9}$$

$$Y_j \geq 0; \qquad \sum_{j=1}^{m} Y_j = 1;$$

In this case we are lead to the idea of a random or mixed strategy. For a bi-matrix game, it is difficult to define a solution that is simultaneously the optimum for both players. We can, however, to define an equilibrium stable set of strategies. A stable solution has the feature that, given choice X_{ij} of player *I*, player *II* is not motivated to change his probabilities Y_{ij}. Likewise, given Y_{ij}, player *I* is not motivated to change X_{ij}. Such a solution, where no player is motivated to unilaterally change his strategy, is sometimes also known as Nash equilibrium [4]. There may be bi-matrix games with several stable solutions. When the non-constant sum games have multiple or alternative stable solutions we should really look at all of them and take into account other considerations in addition to the loss matrix.

If the expected loss to player *I* is v_I and to player *II* – is v_{II}, therefore the solution of the game is:

$$\sum_{j=1}^{m} a_{ij} Y_i \geq v_I \ ; i = 1, \ldots n;$$

$$\sum_{i=1}^{n} b_{ij} X_j \geq v_{II} \ ; j = 1, \ldots m; \tag{10}$$

$$\sum_{i=1}^{n} X_i = 1;$$

$$\sum_{j=1}^{m} Y_j = 1;$$

In other cases the real situation supposes more participants in the conflict or mission. Then N-players game theory can be used in modeling the decision strategy. Usually, total benefit increase if the players cooperate. In these non-constant sum games the difficulty then becomes one of deciding how these additional benefits due to cooperation should be distributed among the players. The linear programming can provide some style in selecting an acceptable allocation of benefits. But this problem is not a subject of the present examination.

Applications

The core of the planning and decision making process model is based on the above-mentioned ideas from game theory. Game theory was chosen as the starting point because the theory addresses one of the central elements of the process, namely the analysis of opposing courses of action.

The planners on each side of the conflict have a separate (and generally different) payoff matrix, representing each planner's perception of the possible courses of action open to himself and his opponent, and the consequences of the interactions between them.

The essence of the deliberate planning model [5] is the analysis, by the planner, of this payoff matrix and the selection of a single course of action, that is, in some sense, the 'best' one to take, given the perceived options open to the enemy. The selection of a course of action is the command decision and is the key output of the deliberate planning process model.

There are several different ways of defining the 'best' course of action, depending on the criteria used to measure 'bestness'. Four such 'decision' criteria are the criterion of pessimism (maximin), the criterion of optimism (maximax), and the criterion of least regret and the criterion of rationality.

The application of LINGO-software gives the possibility to generate many various experiments and to obtain different results. That sort of research is very useful in the extraction of the experience from the historical data. Thus outline practical and rapid application of game theoretic approaches, applied to contemporary asymmetric conflicts.

Examples

In this examination were applied the game theoretic models proposed above to simulate some real situations [6]. Several tasks were solved based on the LINGO-software illustrating the usefulness of this commercial of the self (COTS) product for the military investigation purposes.

1) Side A organizes an air attack against an object, defended of the side B. There are three ways for side B to implement air defence of an object: Q_1 – the air defence equipment has a ring location; Q_2 – the air defence equipment is centralized in one sector; Q_3 – the air defence equipment is located as a semicircle. The effectiveness criterion is the probability of the achievement of the target by the aircrafts. The matrix of probabilities is given. Define the way to organize the attack of side A to defeat the object with greatest probability and the way to defence of the side B to protect the object from the attack.

B A	Q1	Q2	Q3
P1	0.8	0.2	0.5
P2	0.7	0.3	0.4
P3	0.5	0.6	0.3

```
MODEL:
MIN = LB;
a + b + c = 1;
-LB + 0.8*a + 0.2*b + 0.5*c <= 0;
-LB + 0.7*a + 0.3*b + 0.4*c <= 0;
-LB + 0.5*a + 0.6*b + 0.3*c <= 0;
END
LB        0.4000000
A         0.0000000
B         0.3333333
C         0.6666667
```

```
MODEL:
MAX = PG;
a + b + c = 1;
-PG + 0.8*a + 0.7*b + 0.5*c >= 0;
-PG + 0.2*a + 0.3*b + 0.6*c >= 0;
-PG + 0.5*a + 0.4*b + 0.3*c >= 0;
END
LB        0.4000000
A         0.5000000
B         0.0000000
C         0.5000000
```

$X^* = (0.5, 0, 0.5)$; $Y^* = (0, 0.33, 0.67)$; $v = 0.4$

2) Air Forces Staff has available supplies of three types of Chemical Air Bombs (CAB). During the use of chemical weapon the people are disposed into three types of shelters and when there is an alarm signal they can use only individual equipment for protection. The data for the human losses are given. Define the correlation of CAB in which will have the maximum damages (in %).

Type CAB	1	2	3
I	30	40	35
II	40	20	25
III	25	35	30

```
MODEL:
MAX = PG
GMA+GMB+GMC = 1;
-PG +30*GMA + 40*GMB + 25*GMC >= 0;
-PG +40*GMA + 20*GMB + 35*GMC >= 0
-PG +35*GMA + 25*GMB + 30*GMC >= 0;
END

PG      32.500000
GMA     0.7500000
GMB     0.2500000
GMC     0.0000000
```

X* = (0.75, 0.25, 0); $v = 32.5(\%)$

3) The Division Staff plans to regroup the troops at a new region. They can realize this movement using three different routes. During the movement the enemy could attack only one of four discovered objects. The routes cross the traces of the radioactive pollution. The assessment of the probable human losses (in percents) is given. The troops must be allocated according to the routes in the way to sustain minimal losses (in %).

Objects Routes	1	2	3	4
1	15	14	40	35
2	20	18	30	24
3	40	35	25	20

```
MODEL:
MIN = PG;
GMA+GMB+GMC = 1;
-PG + 15*GMA + 20*GMB + 40*GMC <= 0;
-PG + 14*GMA + 18*GMB + 35*GMC <= 0;
-PG + 40*GMA + 30*GMB + 25*GMC <= 0;
-PG + 35*GMA + 24*GMB + 20*GMC <= 0;
END

PG              28.00000
GMA             0.0000000
GMB             0.6000000
GMC             0.4000000
```

X* = (0, 0.6, 0.4); $v = 28(\%)$

4) The “reds” is given the task to capture the hill 5. The “reds” vanguard consists of three tank battalions with mobile infantry units. The “blues” have fortifications on the East Side of the hill and they defend the position through the mobile infantry and the air force units.

The “reds” can attack the object from three different directions – Northern, Southern and Eastern. The “reds” possible strategies are: *P1* – the attack from north, *P2* – the attack from South, and *P3* – the frontal attack.

The “blues” strategies are *Q1* –the air-force attack; *Q2* – shells; *Q3* – the attack with all weapons; *Q4* – attack with all weapons without the air force.

A	Q1	Q2	Q3	Q4
P1	0.3	0.8	0.7	0.65
P2	0.9	0.01	0.9	0.85
P3	0.6	0.6	0.15	0.25

B	Q1	Q2	Q3	Q4
P1	0.6	0.4	0.4	0.5
P2	0.2	0.85	0.2	0.35
P3	0.7	0.6	0.85	0.85

```
MODEL:
SETS:
OPTA/1..4/: PA, SLKA, NOTUA, COSA;
OPTB/1..3/: PB, SLKB, NOTUB, COSB;
BXA( OPTB, OPTA): C2A, C2B;
ENDSETS

DATA:
C2A =   0.3 0.8 0.7 0.65
        0.9 0.01 0.9 0.85
        0.6 0.6 0.15 0.25
C2B =   0.6 0.4 0.4 0.5
        0.2 0.85 0.2 0.35
        0.7 0.6 0.85 0.85
ENDDATA
```

```
CBSTA    0.5158273
CBSTB    0.5058824
PA( 1)   0.5294118
PA( 2)   0.4705882
PA( 3)   0.0000000
PA( 4)   0.0000000

PB( 1)   0.6402878
PB( 2)   0.3597122
PB( 3)   0.0000000
```

$X^* = (0.64, 0.36, 0)$; $Y^* = (0.53, 0.47, 0, 0)$; $v = 0.51$;

CONCLUSIONS

There are some particular areas in need of development if game theory is to be usefully applied as a tool in wargaming the asymmetric environment. There are some areas but progress in these would go a long way toward the realization of game theoretic war gaming.

(1) Synthesizing the game from the situation and historical data.

The analysts need a suitable tool for automatically enumerate relevant players, their options, and estimated payoffs. It is necessary to create and maintain a database, and to combine the expert knowledge.

(2) Finding and applying optimal strategies.

Multi player games model effectively conditions of contemporary conflicts – coalition creation, transnational organizations, etc. Variety of models corresponds to static or dynamic equilibrium. The strategy improvement bases on the use of expert knowledge of psychological factors.

(3) Directed modification of the game.

To update the games to similar situations has an important meaning for reusing the previous expert assessments on payoffs and previous solutions strategies.

(4) Use of modern modeling software.

Modeling languages give up powerful tools to model the conflict situations through the game theory application. The strategies are experimented and the solutions are proposed to planners and decision-makers.

The problem is whether suggested representation of war games generates emergent collective behavior that resembles realistic military environment. The assumption of complete information is the greatest impediment to the practical application of classic game theory. An asymmetric information game where players have incomplete information on either payoffs or options or both is much more typical of the real world situation. Preliminary results are encouraging and the next experience will present the validation of models. The research is still a work-in-progress.

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Report Goals

- Consider the Asymmetric Threats
- Outline Wargaming Development
- Focus on the Game Theory
- Point to the Modern Software

Description

Asymmetric Threats Aspects

- Politics
- Conflicts
- Terrorism
- Technology

Strategy

Wargaming Development Perspectives

- Joint & Coalition Operations Simulations
- Authentic & Accurate Asymmetric Scenarios
- OOTW Analysis & Prediction

Methodology

Game Theory Models

- Antagonistic Games
- Bi-matrix Games
- Coalition Games

Some Theory

$? = < X, Y, H >$ (1)

$H = [h_{ij}], h_{ij} = H(i,j), 1 \leq i \leq n; 1 \leq j \leq m$ (2)

$E_1(X,y_j) = \sum h_{ij}x_i = const;$ (3)

$E_2(x_i,Y) = \sum h_{ij}y_j = const;$

$\sum x_i = 1; \sum y_j = 1;$ (4)

$E(X,Y) = \sum \sum h_{ij}x_i y_j$ (5)

$[X^*, Y^*, \upsilon] ; \upsilon = E(X^*, Y^*)$ (6)

Some Theory *(cont.)*

- $$H = \begin{matrix} p_1 & r_{12}+p_2 & \dots & r_{1n}+p_n \\ r_{21}+p_2 & p_2 & \dots & r_{2n}+p_n \\ \dots & & & \\ r_{n1}+p_1 & r_{n2}+p_2 & \dots & p_n \end{matrix} \qquad (7)$$

pj >0 are the salvage charges (*j*-strategy of player II)

rij >0 are the losses of player II caused by *i*-action of player I

- $A = [a_{ij}],\ a_{ij} = A(i,j),$ (8)

  $B = [b_{ij}],\ b_{ij} = B(i,j),\ 1 \le i \le n;\ 1 \le j \le m$

  $\sum X_i = 1;\ \sum Y_j = 1;$ (9)

  $X_i \ge 0 \quad Y_j \ge 0$

Some Theory *(cont.)*

The solution is:

$$\sum_j a_{ij} Y_i \geq \upsilon_{\mathbf{I}} \;, \qquad 1 \leq \; i \leq n;$$

$$\sum_i b_{ij} X_j \geq \upsilon_{\mathbf{II}} \;, \qquad 1 \leq j \leq m; \qquad (10)$$

$$\sum X_i = 1; \sum Y_j = 1;$$

Applications

- Analysis of Courses of Action
- Planning Models
- Predictions
- Decision-Making Process

Technology

LINGO Solver

- Two-person games
- Two-person non-constant sum games
- Bi-matrix games with several solutions
- Non-constant sum games with N-players

Conclusions

- Outline the Conflict Situation
- Define the Game Model
- Assign the Assessment Criteria
- Acquire the Game Matrix Data
- Select the Solution

Directions

- Develop the games from the situation and historical data
- Find and apply optimal strategies
- Modify and reuse the games
- Use modern modeling software